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Heavy quark symmetry: ideas and applications

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Abstract. This report is a combined version of two talks presented by the authors at the Edinburgh b-physics Workshop, December 1991. It presents the ideas of heavy quark symmetry and gives an introduction to some applications. The references indicate where to go for more information: they are not intended to be complete, nor do they necessarily refer to the original work on any particular subject.

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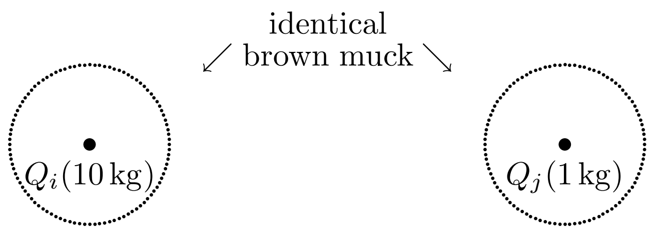
1. The basic idea: no subtleties

Consider two hadrons each containing a single heavy quark. To make our point, let the heavy quark in the first hadron, Q_i, have a mass of 10 kg and let the other quark Q_j, have a mass of 1 kg.



As seen in their rest frames, the hadronic systems that can be built on each heavy quark out of the light degrees of freedom of QCD will be identical: we are just looking at how QCD distributes the "brown muck" of light quarks and glue around a static colour charge. Since the scale of the interactions of the brown muck is set by Λ_{QCD} we can say that when

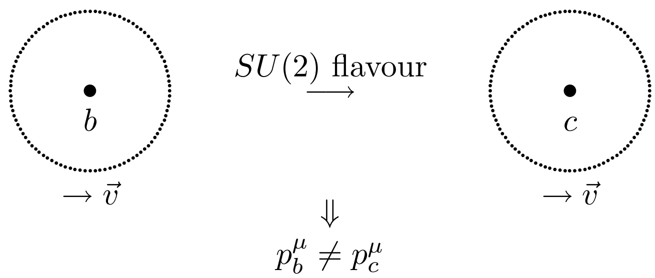
$$m_{Q_i},\, m_{Q_j} \gg \Lambda_{\mathrm{QCD}}$$

the heavy quark will in fact be effectively static so that the light degrees of freedom will be independent of the heavy flavour. For N_h flavours of heavy quark there will be an $SU(N_h)$ symmetry [1] [2]. This is *not* a model: the solution of the QCD field equations will be independent of m_Q as $m_Q \to \infty$. The symmetry is analogous to the isotope effect in atomic physics — the brown muck is independent of the heavy quark mass just as the electronic structure of an atom is independent of the number of neutrons in the nucleus.

We can compare this new symmetry to the ordinary $SU(3)$ flavour symmetry of the light quarks u, d, and s which arises since the light quark masses are *small* compared to Λ_{QCD}. In each case one must be careful to apply the symmetry to the appropriate observables. For example, since the light flavour symmetry arises from the fact that the light quark masses are near the chiral limit, and not because they are nearly degenerate, it *does not* imply that pions and kaons have the same mass. In contrast, the light flavour symmetry *does* imply that the baryons built out of light quarks are approximately degenerate. The pion and kaon are pseudogoldstone bosons whose masses vanish as the quark masses vanish, whereas the baryon masses have a finite limiting value. Similarly, the new symmetry among the heavy quarks, c, b and t arises because they are much *heavier* than Λ_{QCD}. If the b and c quarks weighed 10 kg and 1 kg, the brown muck distributed around them would obviously look the same. The heavy quark symmetry doesn't say that hadrons containing a single b or c quark have the same mass (these masses don't approach a finite value in the heavy quark limit), but it does say that if you line up the lowest energy states (at around 10 kg and 1 kg respectively), the spectra will then match.

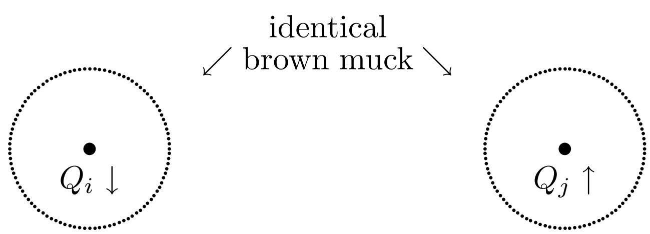
The strange quark mass is actually not very small compared to Λ_{QCD}, so there are sizeable corrections to the predictions of light quark symmetry (about 30%). Likewise, the deviations from heavy quark symmetry will be biggest for the charm quark whose mass is not extremely large compared to Λ_{QCD}: one would expect deviations of about $\Lambda_{\mathrm{QCD}}/m_Q$, or 10% for charm quarks and 3% for b quarks. Heavy quark symmetry would be best for the top quark, but it decays so fast via weak interactions that we do not expect to see top hadrons.

The new symmetry is of an unfamiliar kind. In the two flavour example above we saw the symmetry when both heavy quarks were at rest. By boosting we can see that the heavy flavour symmetry will apply between any heavy quarks of the same *velocity*, not the same momentum.



That is, the $SU(N_h)$ maps $|H_{Q_i}(\vec{v},\lambda)\rangle \leftrightarrow |H_{Q_j}(\vec{v},\lambda)\rangle$. As a result, heavy quark symmetry is a symmetry of certain matrix elements, not a symmetry of the S-matrix. For example, the symmetry can relate form factors in spacelike regions to those in timelike regions.

In fact there is much *more* symmetry than we have mentioned so far. Since the heavy quark spin decouples like $1/m_Q$, just as in atomic physics, in the heavy quark limit the brown muck doesn't care about the spin:



The $SU(N_h)$ flavour symmetry becomes an $SU(2N_h)$ spin-flavour symmetry. This is reminiscent of the old $SU(6)$ quark model, but this time it is exact in the $m_Q \to \infty$ limit. The $SU(2N_h)$ is also like Wigner's $SU(4)$ in nuclear physics.

1.1. Spectroscopy

Since the spin of the heavy quark decouples, we should find states occurring in doublets corresponding to the two possible orientations of the heavy quark spin. We can classify states using the heavy quark spin $\vec{S}_Q$ and the remaining "spin" (combined spin and orbital angular momentum) of the brown muck, which are both good quantum numbers, and which combine to make the total spin of the state,

$$\vec{S} = \vec{S}_Q + \vec{S}_\ell.$$

Hence we should find degenerate doublets characterised by the spin s_ℓ of the brown muck, with total spin $s_\ell \pm \frac{1}{2}$ (unless, of course, $s_\ell = 0$). Of course, heavy quark symmetry can't tell us which s_ℓ quantum numbers will be associated with which states in the spectrum: this is a dynamical issue. In nature we observe (as predicted by the naive quark model) that the lowest-lying mesons with $Q\bar{q}$ quantum numbers have $s_\ell^{\pi_\ell} = \frac{1}{2}^-$ (*i.e.*, the spin and parity of an antiquark). Therefore the degenerate ground state mesons have $J^P = 0^-$ and 1^- and are the B and B^* or D and D^* mesons. Similarly, for baryons we observe, as expected in the quark model, that the lightest states with Qqq quantum numbers have zero light spin ($s_\ell^{\pi_\ell} = 0+$) giving the Λ_b or Λ_c baryons, whilst light spin of one ($s_\ell^{\pi_\ell} = 1^+$) gives the degenerate Σ_Q and Σ_Q^* baryons with $J^P = \frac{1}{2}^+$ or $\frac{3}{2}^+$. These predictions don't depend on a valence quark approximation, or the assumption that s_ℓ is dominated by the light quark spins, but they *do* depend on the identification of the s_ℓ multiplets with the physical states.

As mentioned above, the heavy flavour symmetry tells us that if we line up the ground states, corresponding to subtracting the mass of the heavy quark, then the spectra built on different flavours of heavy quark should look the same. The splittings are flavour independent, although the overall scale is not. This is illustrated in figure 1.

The four strong transitions between any two pairs of doubly degenerate states, occurring via the emission of light hadrons, will be related just by Clebsch-Gordan coefficients. For example, the following factors relate transitions from D_1 and D_2^* to $D\pi$ and $D^*\pi$ states:

			relative coefficient				relative coefficient
D_2^*	$\rightarrow$	$D\pi$	$\sqrt{2/5}$	D_1	$\rightarrow$	$D\pi$	0
		$[D^*\pi]_S$	0			$[D^*\pi]_S$	0
		$[D^*\pi]_D$	$\sqrt{3/5}$			$[D^*\pi]_D$	1

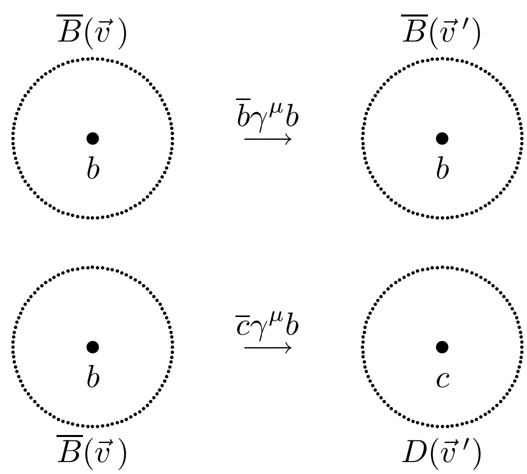
The double degeneracy of the states is lifted at order $1/m_Q$ where the first spin dependence operates. The prediction is that the splitting is $1/m_Q$ times a function at most logarithmic in m_Q. For the vector and pseudoscalar mesons, if you approximate m_Q by the average, $(m_V + m_P)/2$, you predict $m_V^2 - m_P^2$ should be roughly constant. Experimentally, this is very well satisfied for B, D and even K mesons.

$$\begin{aligned} m_{B^*}^2 - m_B^2 &= 0.55\,\mathrm{GeV}^2 \\ m_{D^*}^2 - m_D^2 &= 0.56\,\mathrm{GeV}^2 \\ m_{K^*}^2 - m_K^2 &= 0.53\,\mathrm{GeV}^2 \end{aligned}$$

Initial lattice calculations have been done [3] to find the B–B^* splitting using heavy quark methods. However, even including a perturbative matching correction [4] to get from the lattice result to the continuum value, the result, $(0.19 \pm 0.04 - 0.07)\,\mathrm{GeV}^2$, is still about one third of the experimental value.

1.2. Current matrix elements

Consider the matrix element of the b-number current between a $\overline{B}$ meson of velocity $\vec{v}$ and one of velocity $\vec{v}\,'$. Compare this to the matrix element of the current $\bar{c}\gamma^\mu b$ between a $\overline{B}$ of velocity $\vec{v}$ and a D meson of velocity $\vec{v}\,'$.



The heavy flavour symmetry says the brown muck doesn't know the difference, since it cares only about the velocity of the colour sources carried by the heavy quarks. In equations:

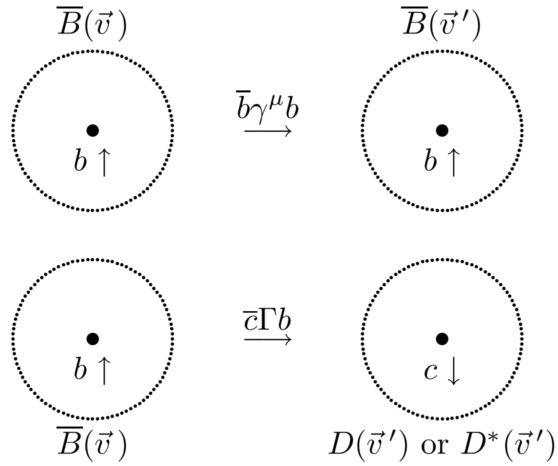
$$\begin{aligned}
\langle \overline{B}(\vec{p}\,'_B)|\,\bar{b}\gamma^\mu b\,|\overline{B}(\vec{p}\,)\rangle &= F_B(t_{BB})(p+p'_B)^\mu \\
\langle D(\vec{p}\,'_D)|\,\bar{c}\gamma^\mu b\,|\overline{B}(\vec{p}\,)\rangle &= f_+(t_{DB})(p+p'_D)^\mu + f_-(t_{DB})(p-p'_D)^\mu
\end{aligned}$$

where ${p'_X}^\mu = m_X v'^\mu$ and $t_{XB} = (p - p'_X)^2$. The symmetry says that $f_\pm$ are related to F_B. We find, equating coefficients of v and v' and removing the trivial effects of the heavy quark masses from the normalisation of states,

$$f_\pm(t_{DB}) = \frac{m_D \pm m_B}{2\sqrt{m_D m_B}} F_B(t_{BB}),$$

with $t_{DB} = (m_B - m_D)^2 + t_{BB}\, m_D/m_B$. Furthermore, since $\bar{b}\gamma^\mu b$ is a symmetry current, counting b-number, the absolute normalisation of F_B is known at $v = v'$ or $t_{BB} = 0$. Hence we know $f_\pm$ at the "zero-recoil" point $t_{DB} = (m_B - m_D)^2$ where a $\overline{B}$ at rest decays to a D at rest.

The spin symmetry of the heavy quark theory lets us say even more. We can consider the matrix element of a current $\overline{c}\Gamma b$, where Γ is *any* Dirac matrix, between states where the b and c quarks have *any* spin, and relate it to F_B.



F_B contains the non-perturbative information on the response of the brown muck to a change in the velocity of the colour source from v to v'. The spin-flavour symmetry tells us that we can use *any* current to kick the heavy quark and change its velocity (and spin and flavour).

1.3. One subtlety

So far we have been economical with the truth. The preceding arguments apply in a low energy effective theory with a cutoff μ, with

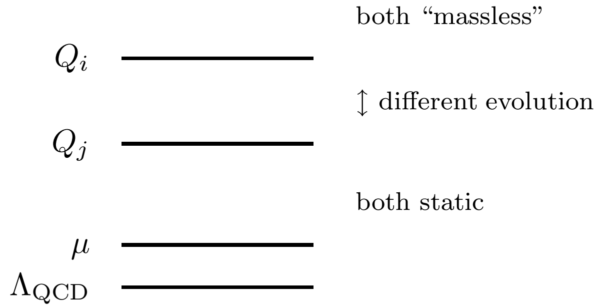
$$\Lambda_{\mathrm{QCD}} \ll \mu \ll m_{Q_j} \le m_{Q_i}.$$

Momenta above m_Q, however, probe non-static heavy quarks Q. The results discussed above applied to the current J in the low energy theory in which momenta larger than μ are cut off. This current is related to the full current j according to

$$j_\nu^{ji} = C_{ji} J_\nu^{ji} + \mathrm{O}(1/m_Q) + \mathrm{O}(\alpha_s/\pi).$$

That is, since the effective theory and the full theory differ at high energy there is a calculable perturbative QCD matching between the full and effective currents. This matching gives a correction factor C_{ji} between the two currents, as well as generating extra operators in the effective theory which match to the full theory current (the α_s/π terms). There are also additional corrections of order $1/m_Q$ for heavy quarks which are not infinitely massive.

If you think of scaling down from very high energy to low energy the picture looks like this:



At scales above the mass of both quarks Q_i and Q_j, the full vector and axial vector currents are partially conserved and so have zero anomalous dimension. The quarks are roughly "massless" and there is no contribution to C_{ji} in this region. Once we come below m_{Q_i}, however, the i quark is regarded as heavy whilst the j quark is not, so in this region the current is not conserved, and C_{ji} is different from unity [5] [6] [7]. Once we move below m_{Q_j}, both quarks are heavy. Again the current is not conserved unless the quarks have the same velocity, in which case they are related by the heavy flavour symmetry. Hence in the low energy region there is a velocity dependent contribution to C_{ji} which reduces to 1 if $v_j = v_i$. We will see this in more detail below.

2. Heavy Quark Effective Field Theory

In this section we describe how the ideas of heavy quark symmetry [1] [2] can be embodied in a low energy heavy quark effective field theory (HQET) [8] [9]. This will allow us to derive Feynman rules for the heavy quarks and give a recipe for doing calculations. See the TASI Summer School lecture notes by Georgi [10] for more details. (Incidentally, several other reviews of heavy quark symmetry and its applications are available: see [11] [12] [13] for example).

First consider a bound state with velocity v_μ, and mass M_Q, which contains a single heavy quark (or antiquark) together with some brown muck. The momentum of the bound state is

$$P^\mu = M_Q v^\mu.$$

For a heavy quark we expect the quark mass m_Q to be nearly equal to the bound state mass, $m_Q \approx M_Q$, with the difference independent of m_Q. We also expect the heavy quark to carry nearly all of the momentum of the bound state, although the brown muck will carry a small momentum q^μ. We can write an equation for the quark momentum, p^μ,

$$p^\mu = P^\mu - q^\mu = m_Q v^\mu + k^\mu,$$

where we define the residual momentum, k^μ,

$$k^\mu = (M_Q - m_Q)v^\mu - q^\mu.$$

The four velocity of the heavy quark is,

$$v_Q^\mu = \frac{p^\mu}{m_Q} = v^\mu + \frac{k^\mu}{m_Q}$$

so that the velocity of the bound state and the quark are the same in the heavy quark limit. As $m_Q \to \infty$ the heavy quark is nearly on-shell and carries nearly all of the bound state's momentum.

The QCD interactions do not change the heavy quark's velocity at all. Any kinks in the trajectory of the heavy quark must be caused by external, *non*-QCD, agencies like weak or electromagnetic interactions.

2.1. $m_Q \to \infty$ in strong interaction diagrams

First look at the spinor u for a heavy quark. Since the final momentum in any strong interaction diagram in the low energy effective theory will differ from the initial momentum by an amount much less than m_Q, the heavy quark spinor satisfies $\not{v}u \simeq u$. Now consider the usual fermion propagator,

$$\frac{i}{\not{p} - m_Q}.$$

Again, let $p^\mu = m_Q v^\mu + k^\mu$ and look at the limit of large m_Q to see that,

$$\begin{aligned} \frac{i}{\not{p} - m_Q} &= \frac{i(\not{p} + m_Q)}{p^2 - m_Q^2} = \frac{i(m_Q \not{v} + \not{k} + m_Q)}{2m_Q v\cdot k + k^2} \\ &\approx \frac{i}{v\cdot k}\frac{1+\not{v}}{2}. \end{aligned}$$

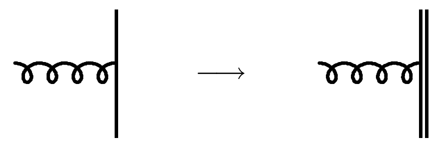
The $(1+\not{v})/2$ projection operator can always be moved to a spinor u satisfying $\not{v}u = u$ (since, as we will see below, $\not{v}$ commutes with the heavy quark-gluon vertex), so we replace the projector by 1. Hence we have a rule for replacing propagators of heavy quarks according to:

$$\frac{i}{\not{p} - m_Q} \quad \longrightarrow \quad \frac{i}{v\cdot k}$$

For the vertex between a heavy quark and a gluon, observe that it will always occur between propagators or on-shell spinors, so we can sandwich it between $(1+\not{v})/2$ projectors, and use

$$\frac{1+\not{v}}{2}\gamma^\mu\frac{1+\not{v}}{2} = v^\mu\frac{1+\not{v}}{2}$$

to obtain the replacement rule:



$$-ig\gamma^\mu \frac{\lambda^a}{2} \qquad\qquad -igv^\mu \frac{\lambda^a}{2}$$

where we have again moved the projection operator to act on a spinor where it gives 1.

The new Feynman rules contain no reference to the heavy quark mass so they show explicitly the symmetry under change of heavy quark flavour. Similarly, there are no γ-matrices in the Feynman rules, so the heavy quark spin symmetry is also apparent.

In the *static* theory, where we expand around $v = (1, 0, 0, 0)$, the propagator and vertex become the nonrelativistic propagator and charge density coupling respectively,

$$\begin{aligned} \frac{i}{v\cdot k} \quad &\rightarrow \quad \frac{i}{k^0} = \frac{i}{E - m_Q}, \\ -igv^\mu \frac{\lambda^a}{2} \quad &\rightarrow \quad -ig\delta^{\mu 0} \frac{\lambda^a}{2}. \end{aligned}$$

2.2. A systematic expansion

We would like to have a low energy effective theory which will allow us to incorporate α_s and $1/m_Q$ corrections systematically [9]. For each velocity v^μ, the Feynman rules above are those arising from a Lagrangian

$$\mathcal{L}_v = i\overline{Q}_v v_\mu D^\mu Q_v$$

where $D^\mu = \partial^\mu - igG^\mu_a \lambda^a/2$ is the colour covariant derivative, and the field Q_v is related to the ordinary heavy quark field Q by

$$Q = \exp(-im_Q v\cdot x) Q_v, \qquad \not{v} Q_v = Q_v.$$

We have done this for the heavy quark only. There is a similar procedure to incorporate the heavy antiquark. The antiquark is different only in being in a conjugate colour representation. There is not enough energy to create heavy quark-antiquark pairs, so the quark and antiquark fields are independent in the HQET (see [10] for more details).

The above applies at lowest order in $1/m_Q$. Corrections suppressed by inverse powers of the heavy quark mass can be straightforwardly incorporated. For example, at dimension five there are two new operators

$$\overline{Q}_v \frac{(iD)^2 - (iv\cdot D)^2}{2m_Q} Q_v, \qquad \overline{Q}_v \frac{i\sigma^{\mu\nu} D_\mu D_\nu}{2m_Q} Q_v,$$

which are the heavy quark kinetic energy and a colour magnetic moment term, respectively. The appearance of the $\sigma^{\mu\nu}$ in the magnetic moment term is the first time anything has distinguished the heavy quark spins. Hence, the magnetic moment operator will be responsible for splitting the vector and pseudoscalar heavy-quark meson masses at order $1/m_Q$.

The underlying spin and flavour symmetries are easy to see in the effective lagrangian,

$$\mathcal{L} = \sum_{\vec{v}} \sum_{j=1}^{N_h} i\overline{Q}_{j_v} v\cdot D Q_{j_v} \, .$$

For each $\vec{v}$ we can rotate any spin component of any flavour of heavy quark into another, giving the $SU(2N_h)$ spin-flavour symmetry. Lorentz transformations mix up the different velocities of heavy quark. The overall symmetry has been christened "$SU(2N_h)^\infty \otimes$ Lorentz" by Georgi.

2.3. Relation to QCD

By construction, the heavy quark effective theory (HQET) described in the two steps above reproduces the low energy behaviour of QCD: we build the heavy theory demanding that it give the same S-matrix elements as QCD. This means that the naive heavy quark limit must be corrected for the effects of high energy virtual processes. For example, the weak flavour changing b to c current is corrected at order α_s in both QCD and the HQET. The difference between these corrections tells us how we must modify the coefficient of the HQET current, as well as possibly introducing new structures in the HQET current, so that the HQET reproduces the physics.

We illustrate for the case of the current $\overline{b}\gamma^\mu c$. The γ^μ in the QCD current is replaced in the HQET current by [14]

$$\gamma^\mu \longrightarrow \mathbb{\Gamma}^\mu = \left(1 + C_0 \frac{\alpha_\mathrm{s}}{\pi}\right) \gamma^\mu + \frac{\alpha_\mathrm{s}}{\pi} \sum_i C_i \Gamma_i^\mu$$

so there is a new strength for the naively matched γ^μ together with new structures Γ_i^μ (such as v^μ and v'^μ, where v and v' are the heavy quark velocities). We say that we "match the low energy approximation to the full theory". Diagrammatically, the two sets of diagrams shown in figure 2 are matched to determine $\mathbb{\Gamma}$ to order α_s (the figure illustrates the case where both quarks are treated as heavy in the effective theory).

If $m_b \gg m_c \gg \mu$, the new coefficient of the naive γ^μ turns out to have the form shown above with

$$C_0 = \ln\frac{m_b}{m_c} - \frac{4}{3}[w\, r(w) - 1] \ln\frac{m_c}{\mu}$$

where $w = v\cdot v'$ and

$$r(w) = \frac{\ln(w + \sqrt{w^2-1}\,)}{\sqrt{w^2-1}} .$$

The leading logs can be summed by the renormalisation group with the result [15] [16],

$$\mathbb{\Gamma}^\mu = C_{cb}\gamma^\mu + \frac{\alpha_s}{\pi}\sum_i C_i\Gamma_i^\mu$$

with

$$C_{cb} = \left[\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right]^{-6/25}\left[\frac{\alpha_s(m_c)}{\alpha_s(\mu)}\right]^{8[w\,r(w)-1]/27} .$$

The μ-dependence in C_{cb} is cancelled by the μ-dependence of the non-perturbative function describing the brown muck transition from v to v'. The two factors in C_{cb} correspond to the renormalisation of the current between m_b and m_c and then between m_c and μ, respectively. As advertised earlier, we see explicitly that the second factor is 1 when $w = 1$, in which case the heavy flavour symmetry relates the heavy b and c quarks, so that the current is conserved and has no anomalous dimension. Strictly speaking, if we perform the matching at order α_s we should use the two loop anomalous dimension for the renormalisation group scaling. Then we match once at the b quark scale, where the b quark becomes heavy, scale between m_b and m_c and match again at m_c where the c quark becomes heavy. The anomalous dimension calculations have now been taken to two loops [17] [18], so the matching and scaling can be done.

3. Some Applications

Whenever a symmetry can be identified it gives us calculational power. By relating various matrix elements, and fixing the absolute normalisation of some, heavy quark symmetry enhances our predictive ability, allowing us in some cases to finesse the difficulties of understanding hadronic structure. This is analogous to pion, kaon, and eta physics where the effective chiral lagrangian can be used to extract systematically the consequences of the pattern of chiral symmetry breaking. For heavy quarks we have identified a new symmetry and have developed the heavy quark effective theory to calculate its predictions.

Heavy quark ideas also help us to model b quarks in lattice calculations, giving access to the actual values of further matrix elements. The problem with putting b quarks on the lattice by conventional methods is that their Compton wavelength is smaller than the lattice spacing, and the b quarks "fall through". Heavy quark symmetry allows us to extract the b mass dependence, leaving an effective theory without such a large mass scale, which can be modelled on present day lattices. These ideas were discussed elsewhere at this workshop, and we refer readers to the lattice group's contributions. Here we will concentrate on one area of great potential for heavy quark ideas: constraining the CKM matrix using b-physics.

3.1. *Determining V_{cb}*

One of the first applications of the ideas of heavy quark symmetry was to semileptonic $\overline{B}$ meson decays and the extraction of the b to c mixing angle [19] [1] [2].

V_{cb} can be determined from semileptonic decays of $\overline{B}$ mesons to D and D^* mesons. There are altogether six form factors in the two decays,

$$\begin{aligned}
\langle D(p')| V^\mu |\overline{B}(p)\rangle &= f_+(t_{DB})(p+p')^\mu + f_-(t_{DB})(p-p')^\mu, \\
\langle D^*(p',\epsilon)| A^\mu |\overline{B}(p)\rangle &= f(t_{DB})\epsilon^{*\mu} + a_+(t_{DB})\epsilon^*{\cdot}p(p+p')^\mu + a_-(t_{DB})\epsilon^*{\cdot}p(p-p')^\mu, \\
\langle D^*(p',\epsilon)| V^\mu |\overline{B}(p)\rangle &= ig(t_{DB})\epsilon^{\mu\nu\lambda\sigma}\epsilon^*_\nu(p+p')_\lambda(p-p')_\sigma .
\end{aligned}$$

In these equations, $V^\mu = \overline{c}\gamma^\mu b$, $A^\mu = \overline{c}\gamma^\mu\gamma_5 b$ and $t_{DB} = (p-p')^2$. In section 1.2 we saw how the heavy flavour symmetry related the vector current matrix element to that of the b-number current. Now the spin symmetry relates *all* the $\overline{B} \to D$ and $\overline{B} \to D^*$ matrix elements, so they can each be expressed in terms of *one* universal function $\xi(w)$, where $w = v{\cdot}v'$. This function is the *same* for any heavy quark transition, $Q_i \to Q_j$ with the same brown muck. It describes the response of the brown muck to the change in velocity of the colour source from v to v'. Furthermore, when $v = v'$ there is a flavour symmetry between the two heavy quarks. Then the current causing the transition is a symmetry current so the normalisation of $\xi(w)$ is fixed at the point $w = 1$ which is maximum t_{DB}. We can take the normalisation to be $\xi(1) = 1$. For $\overline{B}$ to $D^{(*)}$ decays this point is the "zero recoil" point where a $\overline{B}$ at rest decays to a D or D^* at rest.

The relations of the form factors to ξ come out as follows:

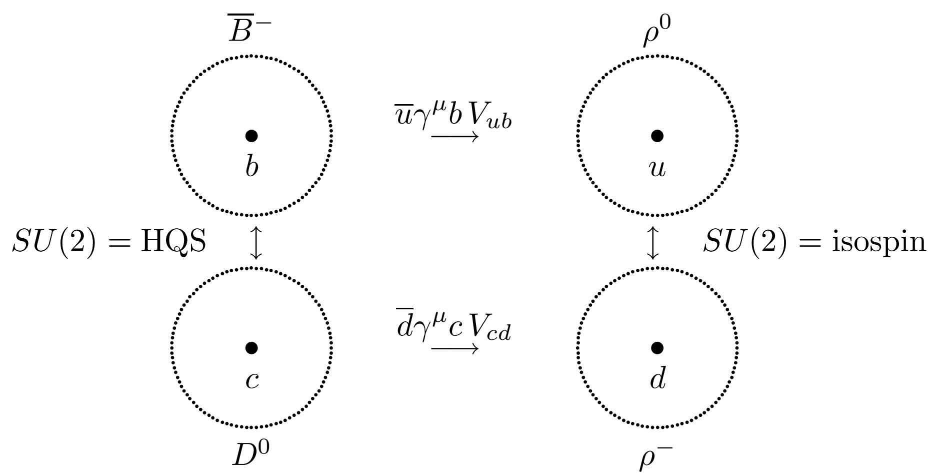
$$\begin{aligned}
f_\pm &= C_{cb}\,\frac{m_D \pm m_B}{2\sqrt{m_B m_D}}\,\xi(w), \\
g = a_+ = -a_- &= C_{cb}\,\frac{1}{2\sqrt{m_B m_D}}\,\xi(w), \\
f &= C_{cb}(w+1)\sqrt{m_B m_D}\,\xi(w).
\end{aligned}$$

The factor C_{cb} is the perturbatively calculable correction to the strength of the current in the HQET that we described above. For details see [16].

If experimental measurements can be reliably extrapolated to the zero recoil point then we can determine the CKM matrix element V_{cb}. Alternatively, we can actually calculate $\xi(w)$ on the lattice, by looking at the $\overline{B}$ to $D^{(*)}$ matrix element as a function of the velocity transfer between the heavy quarks.

3.2. *Determining V_{ub}*

The idea behind using heavy quark symmetry to extract the $b \to u$ mixing angle is illustrated by the following picture [20]:



The heavy quark symmetry relates $\overline{B} \to \rho^0$ to the unphysical $D \to \rho^0$ matrix elements. However, light quark isospin relates ρ^0 and ρ^-, and the decay $D^0 \to \rho^-$ is determined by the known CKM matrix element V_{cd}. This means that we can close the loop in the diagram above to fix V_{ub} from the $\overline{B}^- \to \rho^0$ decay. Note that the two ρ states on the right hand side above are "pure brown muck": the heavy quark symmetry relates matrix elements for b and c quarks surrounded by the same brown muck to go to the same pure brown muck states.

The matrix elements of the vector and axial vector weak currents between $\overline{B}$ or D and π or ρ involve six form factors (defined like those in $\overline{B} \to D^{(*)}$ decay). Since the heavy quark symmetry relates states of the same velocity, one difficulty is that the allowed range of momentum transfer in the D decays does not allow us to cover the whole range for the $\overline{B}$ decays of interest. In its purest form, the above extraction must therefore be done in the region of the $\overline{B} \to \rho$ Dalitz plot where the ρ momenta do not exceed those available in the $D \to \rho$ Dalitz plot. Another potential difficulty may be illustrated by the example of the f_+ form factor. The value of this form factor in $\overline{B}$ decay is determined by *both* f_+ and f_- for D decay, but the f_- contribution to the D decay rate is suppressed by a factor of $(m_\ell/m_D)^2$, where m_ℓ is the mass of a charged lepton. Since the decay to the τ lepton is not kinematically allowed, f_- will be very difficult to obtain from D decay. Fortunately, one can show that in the heavy quark limit there are enough relationships amongst heavy-to-light form factors to overcome this lack of data [20]. In this case, $f_- = -f_+$ up to corrections of order Λ_{QCD}/m_c.

3.3. Rare B decays

Rare B decays are expected to be a good probe of new physics, but if we are to see new physics we had better know the standard model expectation first. HQET can help us [20] by relating the matrix elements of interest, such as in $\overline{B} \to \overline{K} e^+ e^-$, to more easily measured processes like $D \to K^- e^+ \nu_e$. In fact for these examples we need the matrix element of the vector current between the heavy and light pseudoscalar meson states. Using light quark flavour symmetry, this is the same problem as looking at $\overline{B} \to \pi$ or $D \to \pi$ matrix elements considered in section 3.2.

The rare decays $\overline{B} \to K^* \gamma$ and $\overline{B} \to K^* e^+ e^-$ have contributions from a transition magnetic moment operator,

$$\overline{s}_L \sigma^{\mu\nu} b_R.$$

We can use the heavy quark spin symmetry to relate the matrix elements of this operator to those of a current. Set $\mu = 0$ and $\nu = i$ and observe that,

$$\sigma^{0i} \propto [\gamma^0, \gamma^i] = -2\gamma^i \gamma^0.$$

In the rest frame of the heavy b quark, $\gamma^0 b = b$, so, in the rest frame we find,

$$\overline{s}_L \sigma^{\mu\nu} b \quad \overset{\text{related to}}{\longrightarrow} \quad \overline{s}_L \gamma^i b_L.$$

Furthermore, the heavy flavour symmetry allows us to relate the current to $\overline{s}_L \gamma^i c_L$, so that $\overline{B} \to K^* \gamma$ is related to $D^0 \to K^- e^+ \nu_e$. One problem here is that in $\overline{B} \to K^* \gamma$, the K^* has a fixed momentum outside the kinematic range of the corresponding kaon in the semileptonic D decay (this was discussed at the workshop by A. Ali).

3.4. Heavy baryon weak decays

States in which the brown muck has spin $s_\ell = 0$ form spin-1/2 baryons. The heavy quark spin symmetry relates up and down spin states of the baryon, so it will relate baryon form factors among themselves.

As mentioned in section 1.1, the lowest-lying Qqq state containing a single heavy quark is expected to be a Λ_Q with $s_\ell^{\pi_\ell} = 0^+$. The simplest example is the Λ where a strange quark is bound to an isospin zero $s_\ell^{\pi_\ell} = 0^+$ light quark state. The Λ_c has been observed with a mass of 2285 MeV, but the corresponding Λ_b is yet to be confirmed [21] [22]. Heavy quark symmetry tells us that the mass splitting of the pseudoscalar meson and the baryon is independent of the heavy quark mass in leading order in the HQET, so we expect $m_{\Lambda_b} = m_B + m_{\Lambda_c} - m_D$.

The rate for the semileptonic decay $\Lambda_b \to \Lambda_c e \overline{\nu}$ is given in terms of six form factors:

$$\begin{aligned}
\left\langle \Lambda_{Q_j}(p', s') \right| V^\mu \left| \Lambda_{Q_i}(p, s) \right\rangle &= \overline{u}^{s'}(p') \left(F_1^{ji} \gamma^\mu + F_2^{ji} v^\mu + F_3^{ji} v'^\mu \right) u^s(p) \\
\left\langle \Lambda_{Q_j}(p', s') \right| A^\mu \left| \Lambda_{Q_i}(p, s) \right\rangle &= \overline{u}^{s'}(p') \left(G_1^{ji} \gamma^\mu \gamma^5 + G_2^{ji} v^\mu \gamma^5 + G_3^{ji} v'^\mu \gamma^5 \right) u^s(p)
\end{aligned}$$

where $p = m_{Q_i} v$ and $p' = m_{Q_j} v'$. Heavy quark symmetry implies that,

$$F_1^{ji} = G_1^{ji} = C_{ji}\, \eta(w)$$

where C_{ji} is the same renormalisation factor we discussed earlier, arising from matching the HQET to QCD: it depends only on the heavy quarks. The function $\eta(w)$ is a universal (brown muck dependent) function of the velocity transfer, $w = v{\cdot}v'$. The remaining form factors are zero in the heavy quark limit.

Heavy quark symmetry makes the same prediction for decays of Qsu and Qsd baryons, $\Xi_b \to \Xi_c e\overline{\nu}$, and similar predictions for the decay of Qss baryons,

$$\Omega_b \to \Omega_c e\overline{\nu},\ \Omega_c^* e\overline{\nu}.$$

One can also prove that the following decays are forbidden in the heavy quark limit (Σ_Q is a Quu or Qdd baryon):

$$\begin{array}{ccc} \Lambda_b \to \Sigma_c e\overline{\nu} & & \Xi_b \to \Xi_c' e\overline{\nu} \\ & \text{and} & \\ \Lambda_b \to \Sigma_c^* e\overline{\nu} & & \Xi_b \to \Xi_c^* e\overline{\nu} \end{array}$$

In each case we find a common QCD correction to the matrix element, determined by the renormalisation of the current which changes b into c, together with a function of $w = v{\cdot}v'$ which contains the response of the brown muck [23] [24].

Heavy quark symmetry can be applied to some purely hadronic decays. For example, $\Lambda_b \to \Lambda_c D_s$ can be related to $\Lambda_b \to \Lambda_c D_s^*$, since the underlying process, $b \to c\overline{c}s$ involves three heavy quarks (the c and $\overline{c}$ are independent in the HQET) [25].

3.5. Factorisation

Attempts have long been made to justify factorisation in two body decays of pseudoscalar mesons. Factorisation means that the matrix element for $\overline{B} \to D\pi$, for example, can be separated as,

$$\langle D\pi|\, \overline{d}\gamma_\mu u \overline{c}\gamma^\mu b\, |\overline{B}\rangle \approx \langle D|\, \overline{c}\gamma^\mu b\, |\overline{B}\rangle\, \langle \pi|\, \overline{d}\gamma_\mu u\, |0\rangle\, .$$

This cannot really be true since the two sides have different renormalisation point dependence. However, with some extensions of HQET ideas it has been possible to prove [26] that,

$$\Gamma(\overline{B} \to D\pi) = 6\pi^2 f_\pi^2 A^2\ \frac{\mathrm{d}\Gamma(\overline{B} \to De\overline{\nu})}{\mathrm{d}m_{e\overline{\nu}}^2}\bigg|_{m_{e\overline{\nu}}^2 = m_\pi^2}$$

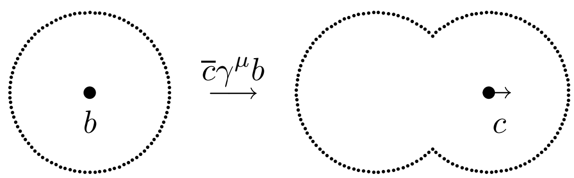
with corrections of order $\Lambda_{\rm QCD}/m_c$. Here $A \approx 1.15$ is a renormalisation group matching factor. This relation agrees well with experiment.

The same method suggests that factorisation need *not* hold in $\overline{B} \to \pi\pi$ or $\overline{B} \to D\overline{D}$, but it does suggest a test: factorisation should hold for $\overline{B} \to D\pi\pi$ when the two final state pions are collinear.

Factorisation and heavy quark symmetry give absolute predictions for ratios of decay rates, for example, $\Gamma(\overline{B} \to D\pi)/\Gamma(\overline{B} \to D^*\pi)$. The order $\alpha_s(m_b)/\pi$ corrections (both factorisable and non factorisable) to this result are small [27], although their precise value is not well determined because of cancellations.

3.6. A Bjorken sum-rule for $\xi(w)$

Consider the following picture in which a b quark with velocity v surrounded by brown muck is kicked by a $\overline{c}\gamma^\mu b$ current, giving a c quark moving at velocity v'. The brown muck must rearrange itself and reform with some surrounding the moving c quark, and some possibly left behind. Clearly we have a set of possible outcomes where the c quark has all possible brown muck configurations reachable from the initial one, each outcome having an associated probability. This will give us a sum rule [28] [29].



Since the brown muck cannot change the heavy quark velocity in the heavy quark limit (the "velocity superselection rule") one can obtain the following sum rule:

$$\mathrm{Rate}[b(\vec{v}\,) \to c(\vec{v}\,')] = \sum_{X_c} \mathrm{Rate}[\overline{B}(\vec{v}\,) \to X_c(\vec{v}\,')].$$

The inclusive rate for a b quark of velocity $\vec{v}$ to go to a c quark of velocity $\vec{v}\,'$ is obtained by summing over all possible states containing a c quark of velocity $\vec{v}\,'$ which can be reached from a $\overline{B}$ containing a b quark of velocity $\vec{v}$. That is, the heavy quark is undisturbed by the "splash" of the brown muck. Explicitly:

$$\begin{aligned}
1 = {} & \frac{w+1}{2}\,|\xi(w)|^2 \\
& + \frac{1}{2}(w-1)^2(w+1) \sum_{n=1}^{n_{\max}(\mu)} |\xi^{(n)}(w)|^2 \\
& + 2(w-1) \sum_{m=1}^{m_{\max}(\mu)} |\tau_{1/2}^{(m)}(w)|^2 \\
& + (w-1)(w+1)^2 \sum_{p=1}^{p_{\max}(\mu)} |\tau_{3/2}^{(p)}(w)|^2 \\
& + \cdots
\end{aligned}$$

The successive lines on the right hand side refer first to the sum over D and D^* followed by the sums over other states with light spin $s_\ell^{\pi_\ell} = \frac{1}{2}^-$, then $s_\ell^{\pi_\ell} = \frac{1}{2}^+$ and

$s_\ell^{\pi_\ell} = \frac{3}{2}^+$ and so on. The heavy quark symmetry makes the RHS doable. For example, each $s_\ell^{\pi_\ell} = \frac{3}{2}^+$ multiplet has eight form factors all related to a single function $\tau_{3/2}$.

The interpretation is that as rate disappears from the "elastic" channel ($D^{(*)}$) as you move away from $w = 1$, it appears in the excited states. This is analogous to the Cabibbo–Radicati sum rule for the proton form factor. If you write the universal function ξ as

$$\xi(w) = 1 - \rho^2(w-1),$$

where ρ is the slope or "charge radius", you can prove that

$$\rho^2 = \frac{1}{4} + \sum \left| \tau_{1/2}^{(m)}(1) \right|^2 + 2 \sum \left| \tau_{3/2}^{(p)}(1) \right|^2 ,$$

i.e., the compensation is only by the $s_\ell^{\pi_\ell} = \frac{1}{2}^+, \frac{3}{2}^+$ states, and $\rho^2 \geq \frac{1}{4}$. In the harmonic oscillator quark model, the compensating states are the lowest excitations of the brown muck, obtained by combining a light quark of spin-$\frac{1}{2}$ with an orbital angular momentum of 1. The $s_\ell^{\pi_\ell} = \frac{3}{2}^+$ states are known and ρ^2 is now measured. This sum rule should soon give an interesting constraint on theory/experiment.

There is an analogous sum rule for the Λ_b [30]:

$$\eta(w) = 1 - \rho_\Lambda^2(w-1) + \cdots$$

where

$$\rho_\Lambda^2 = 0 + \sum \left| \sigma^{(n)}(1) \right|^2 .$$

This time, the one-quarter is replaced by a zero and the sum is over the states with $s_\ell^{\pi_\ell} = 1^-$, which are again the lowest excitations in the quark model.

3.7. $\Lambda_{\mathrm{QCD}}/m_Q$ corrections vanish at $w = 1$

Heavy quark symmetry makes many predictions in the $m_Q \to \infty$ limit. However, since the quarks to which we wish to apply these ideas are not really infinitely massive, we must ask about $\Lambda_{\mathrm{QCD}}/m_Q$ corrections. Fortunately, the effective field theory organises these correction effects, so there is some predictive power from the symmetry even when it is broken [31]. This is reminiscent of the Gell-Mann–Okubo and Coleman–Glashow formulas.

The most surprising case so far is for the form factors in the matrix elements for the heavy baryon decay $\Lambda_b \to \Lambda_c$ [32]. Just one new constant, Δm is required, and the universal function $\eta(w)$ is still normalised, $\eta(1) = 1$. The situation is summarised as follows:

	leading order	with Λ/m_Q corrections
F_1	$\eta(w)$	$\eta(w)(1+\overline{\Delta})$
F_2	0	$-\eta(w)\overline{\Delta}$
F_3	0	0
G_1	$\eta(w)$	$\eta(w)$
G_2	0	$-\eta(w)\overline{\Delta}$
G_3	0	0

The $1/m_Q$ corrections are given in terms of $\overline{\Delta} = \Delta m/[m_c(1+w)]$. Perturbative QCD corrections will add a multiplicative correction factor C_{cb} to all terms, as we discussed above. Then the result is good up to corrections of orders, $(\Delta m/m_c)^2$, $\Delta m/m_b$ (only the c-quark was given a finite mass) and $\alpha_{\rm s}(\Delta m/m_c)$.

The fact that the form factor G_1 retains a known normalisation at zero recoil $w = 1$ offers the possibility of extracting V_{cb} with reasonably small uncertainties. In fact, at zero recoil there are *no* $1/m_Q$ corrections to the matrix elements of the vector and axial vector currents [33]. To understand why, recall that the charges associated with V^μ and A^μ are symmetry generators when $v = v'$, and b quarks can be rotated into c quarks of the same velocity. More explicitly, write the ground state baryon for finite m_Q as a perturbative sum over states in the $m_Q \to \infty$ limit:

$$|\psi^0_{m_Q}\rangle = |\psi^0_\infty\rangle + \sum_{n\neq 0} |\psi^n_\infty\rangle \,\frac{\langle\psi^n_\infty|\,\mathrm{O}(\Delta m/m_Q)\,|\psi^0_\infty\rangle}{(E_n - E_0)}.$$

Since the $m_Q \to \infty$ states are eigenstates of the charges, we have $\langle\psi^0_\infty|\,Q\,|\psi^n_\infty\rangle = \delta^{n0}$, so that,

$$\left\langle\psi^0_{m_2}\right|Q\left|\psi^0_{m_1}\right\rangle = 1 + \mathrm{O}(\Delta m/m_Q)^2.$$

For semileptonic $\overline{B} \to D^{(*)}$ decays things are not so simple. The new constant Δm enters and only two of the six form factors are unaffected by $1/m_Q$ corrections. In particular, the form factor proportional to $v^\mu + v'^\mu$ in the vector current matrix element is unaffected. However, experiments measure a form factor proportional to $p^\mu + p'^\mu = m_b v^\mu + m_c v'^\mu$ (the form factor accompanying $p^\mu - p'^\mu$ picks up a factor of the lepton mass when contracted with the leptonic part of the matrix element for the decay), which contains an admixture of a corrected form factor. Fortunately, at zero recoil, the $\overline{B} \to D^*$ matrix element depends solely on an uncorrected form factor, so this may be the best way to extract V_{cb} from semileptonic $\overline{B}$ decays. Initial analysis by Neubert gives [34],

$$|V_{cb}|\left(\frac{\tau_B}{1.18\,\mathrm{ps}}\right) = 0.045 \pm 0.007$$

which is slightly more precise than values extracted using model dependent analyses, and is *model independent*.

4. Status and prospects

The number of papers on heavy quark symmetry produced in the last two years is in the hundreds. New papers on this subject appear nearly every day. Corrections for finite heavy quark masses and for perturbative QCD matching have been classified and calculated, and some phenomenology done. The absence of $1/m_Q$ corrections at $v = v'$ is possibly the most significant recent development.

It now appears that it may be useful to think of hadrons containing a single heavy quark as the "hydrogen atoms of QCD". There are many advantages to this limit: relativistic effects are simplified and the heavy quark acts as a pointlike probe of the light "constituent quarks". The symmetries and rigorous results of the heavy quark limit can be used for consistency checks (in the form of "boundary conditions") on models.

In a more practical vein, new data from beauty and charm factories should allow us both to test heavy quark symmetry and obtain tighter limits on standard model parameters. Heavy quark ideas applied to lattice calculations may allow the theoretical prediction of strong interaction matrix elements which were unavailable before.

Ironically, heavy quarks may prove to be an essential tool in finally helping us to understand the nature of the brown muck of QCD.

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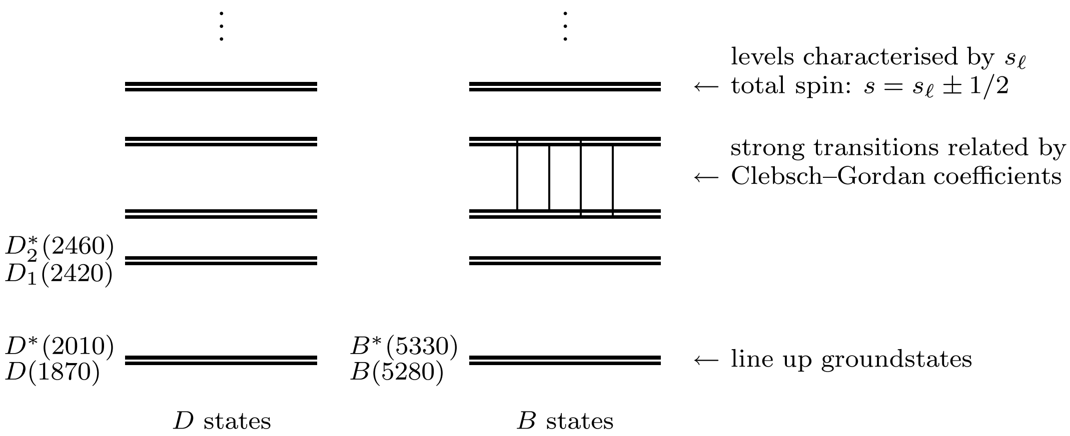

Figure 1. Spectrum of states predicted by heavy quark symmetry

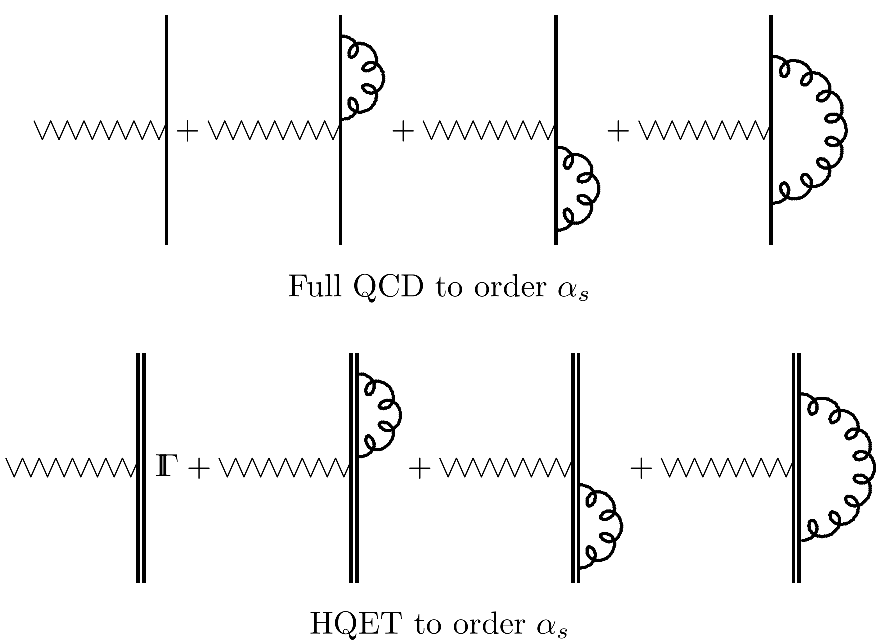

Figure 2. Matching a current between QCD and the heavy quark effective theory